

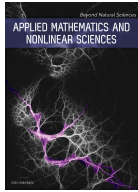
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## Application of Higher Order Ordinary Differential Equation Model in Financial Investment Stock Price Forecast

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### Abstract

In order to improve the modelling efficiency in dynamic system prediction, this paper proposes a predictive model based on high-order normal differential equations to model high-order differential data to obtain an explicit model. The high-order constant differential equation model is reduced, and the numerical method is used to solve the predictive value. The results show that the method realises the synchronisation of model establishment and parameter optimisation, and greatly enhances the modelling efficiency.

**Keywords:** High order constant differential equation model; dynamic system modelling; financial investment; stock price.

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## 1 Introduction

Dynamic systems vary from time to time in daily life, such as temperature changes, precipitation and financial data change. How to model the prediction of dynamic system modelling with time has always been a research hotspot. The appropriate time series model is of great significance for investment risk controlling investment output assessment.

Time series prediction is a method for building a model based on the regular information of existing data, and the model is introduced to complete the prediction method. The prediction effect is mainly affected by the model, which is because time series data is a non-regular dynamic system. The data become complicated due to time volatility, and the different models have a great difference between the processing of data, build

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contacts and regular discovery. The model has a different degree of deviation to the description of historical data, which in turn has a direct impact on the prediction. For dynamic system sampling data over time, the ARIMA model is usually predicted with the artificial neural network (ANN) model. The ARIMA model is a-class linear model that is poorly handled on nonlinear problems. In order to improve its nonlinearity, the literature combines the ARIMA model with the deep belief network, support vector machine and GARCH, and has made a certain amount of red tide forecast, uranium price prediction, network traffic forecasting and subway passenger short-term forecasting effect [1].

At the end of the seventeenth century, the sub-division was accompanied by the development of calculus, born due to the integrity and application of its operation, so that it quickly became a powerful tool for studying natural science. Scientists began discovering that the actual engineering issues in many aspects of nature can be used to establish a sub-equation model with initial value and boundary conditions. Examples are the speed resort differential model established by solving the fastest drop, the Malthus population model and the Logistic model established by the population forecast; the non-uniform beam is of horizontal vibration, and of 6th order, 8th-order, 10th-order normally differential equation model of the ring structure vibration problem. Over a period of time, although scientists have established a large number of solutions to the equation, how to solve these models is an urgent need. The simple model is also good, which can be accurately solved using the direct integral method, separation variable method and so on; however, most models in real life cannot give precise solutions due to the particularity of their physical background complexity and boundary problems. Due to this happening, it has caused scientists to study the solution from other aspects. Some scientists have begun to think that as there is no exact solution for the sub-partition, it would be a good idea to use an approximation to solve it. Based on this idea, the numerical solution of differential equations has been branched, and then it was rapidly developed and it has now become a hot topic in the field of mathematics research [2].

Khachay solved the boundary value problem of equation based on Meyer. In the utilisation of many solutions of solutions, many scholars favour simple forms of solutions. Efendiev studied the Haar function vector and established a HAAR wavelet integrated calculator matrix to provide the basis for using the HAAR wavelet solution differential equation [4]. BAGD applied the HAAR wavelength division operator matrix to the power system problem, and promoted the application of wavelet in the power system [5]. XIE used the HAAR wavelet method to solve the linear sonocity division, nonlinear sub-division, high-order differential equations, one-dimensional diffusion equation, two-dimensional Poisson equation, as well as a change in the variable steps Wavelet method [6]. A cooperation will extend the HAAR wavelet configuration method to linear integral equations, the second type of Freholm nonlinear integral equations, numerical methods for nonlinear solution equations [7]. In China, Karaman used HAAR to solve the wavelet number of wavelet mean for wave equation [8]. KUMAR has obtained a numerical solution of the development equation with slopes HAAR [9]. Kennedy used HAAR to use the eigenvalues of high-order differential equations and three-dimensional parts equations and three-dimensional double-tuning equations in the formal area [10].

## 2 Model establishment and solving based on high-order normal differential equations

### 2.1 Higher order ordinary differential equation model

In the case of known historical data, calculate the differential differences according to the central difference in the literature, then establish the function relationship between the differential value of the next time node and the historical data difference value, such as in Formula (1):

$$\begin{cases} x^{(n)}(t) = f(t, x(t), x'(t), \dots, x^{(n-1)}(t)) \\ x^{(i)}(t_0) = x^{(i)}(0) \end{cases} \quad (1)$$

where  $T$  is the time node,  $x(t)$  represents the stock clip price of the  $T$  node, and  $x'(t), \dots, x^{(n-1)}(t), x^{(n)}(t)$  are the differential differences.

With the GEP algorithm, the display expression of the high-order alternative equation model of each stock can be obtained for subsequent analysis. At the same time, in order to achieve the goal of utilisation of multi-factor prediction, on the basis of standard GEP, other indicators affecting the stock price change are added to the adaptive function, and finally the high-order regular differential equation model based on multi-factor regularisation GEP algorithm is obtained.

In the evolutionary algorithm, the adaptation function is the main indicator described in the individual performance, guiding the evolutionary direction, which can affect the convergence speed of the algorithm and whether the optimal solution can be found. Different complex systems correspond to different adaptive functions. For the stock system, simple assessment is evaluated as adapted, which is easy to cause the predicted effect, and the error is large. The stock price is affected by many factors, and different indicators have different effects on the stock price. Therefore, this paper improves the adaptation function joining the impact indicator, and constrains the share price as a regular item.

### 2.1.1 Improvement of the adaptation function

The standard regularisation theory only involves linear problems, adding constraints for experience error functions. It will constrain a priori knowledge, play a guiding role, and tend to select the direction of gradient decrease in constraints in the process of optimising the error function, so as to ultimately solve the prior knowledge. Simply put, regularisation thinking is to find an approximate solution close to the precise solution to make it as close as possible.

Since the volume of the transaction is one of the indicators of the assessment stock, there is a certain degree of influence on price fluctuations, and this paper adds to the GEP algorithm as a regular item, and thereby the standard GEP is improved.

Because the amount of the volume and the closing price is large, it is not convenient for data analysis, so the transaction amount indicator must first be standardised, and the calculation made to the interval [0, 1] as in Formula (2).

$$v(t) = \frac{\text{volume}(t) - \text{volume}_{\min}(t)}{\text{volume}_{\max}(t) - \text{volume}_{\min}(t)} \quad (2)$$

$$p(t) = x^{(n)}(t)K(v(t)) \quad (3)$$

The regular item is  $\omega \sum |p(t) - p(t-1)|$ ;  $\hat{x}^{(n)}(t)$  is the  $n$ -order value of the predicted stock price,  $x^{(n)}(t)$  is the  $N$ -step Nag value of the actual stock price;  $\omega$  is the weight coefficient of the regular item, reflecting the degree of influence of the transaction index for the stock price; and  $K(V(t))$  is mapping for the subunits. For problems required by this article, the specific value should be better.

The above improvement adaptation function, based on the standard GEP's adaptivity function, the absolute error function, along with the transaction index as the regular item, constrained the stock price forecast, avoiding a large error in using a single indicator prediction. At the same time, the enhancement algorithm jumps out of local optimal capabilities and improves the prediction accuracy. For calculation of the regular item parameters, this paper uses the correlation between the indicators to determine the weight coefficient, and then determines the subunits in the adaptive function based on the basic theory of the fuzzy rough set. Improved adaptation functions are used to measure the advantages and disadvantages of the model while increasing the accuracy of data prediction [11].

## 2.2 Solution of high-order normal differential equation model

### 2.2.1 Determination of weight $\omega$

There are a lot of influencing factors of stock prices, and each indicator is different from the size of the stock price. It is different from the correlation between the stock price, so the weights of each indicator should also be different. This article has the following solving method for the weight factor of the regular item in the adaptive function.

Suppose  $A_j$  indicates that the amount of information included in the  $j$ th indicator, that is, the differentiated information  $Z_j$  is expressed, the correlation coefficient between the  $j$ th indicator and other indicators is  $r_{jk}$ , and the calculation formula of  $r_{jk}$  and  $Z_j$  is known as shown in Formulas (5) and (6):

$$r_{jk} = \frac{\sum (X_j - \bar{X}_j)(X_k - \bar{X}_k)}{\sqrt{\sum (X_j - \bar{X}_j)^2 \sum (X_k - \bar{X}_k)^2}} \tag{4}$$

$$j = 1, 2, \dots, l, k = 1, 2, \dots, l$$

$$Z_j = \frac{S_j}{\bar{X}_j}, j = 1, 2, \dots, l \tag{5}$$

$$\mu = \bar{X}_j = \frac{1}{N} \sum_{p=1}^N x_{pj}, S_j^2 = \frac{1}{N-1} \sum_{p=1}^N (X_{pj} - \mu)^2 \tag{6}$$

Then  $A_j$  can be represented as:

$$A_j = Z_j \sum_{k=1}^l \left( 1 + r_{jk}^2 + \frac{r_{jk}^2}{Z_j - r_{jk}} \right), j = 1, 2, \dots, l \tag{7}$$

where the larger the  $A_j$ , the greater the amount of information contained in the  $J$  IT, and the greater the importance of this indicator, the greater the weight, so the weight of the  $JU$  JII is (8):

$$\omega_j = \frac{A_j}{\sum_{j=1}^l A_j} \tag{8}$$

In this article, the two indicators selected are stock daily closing prices and daily transactions. Therefore, the weight coefficient of the regular item is  $\omega = \omega_2: \omega_1$ .

Thus, by Formulas (4)–(7), the transaction amount indicator is quantified for the importance of the stock price, and the weight coefficient value of the regular item is given for the size of the influence on the stock price, which can be effective. This reduces the effects of extreme values, making the calculation results more reasonable and reliable.

**2.2.2 Sub function k (V (t)) determination**

The fuzzy set theory was proposed by the US computer and control the theory of experts in 1965 and the rough set theory was proposed by Polish mathematician Pawlak in 1982; it is a method of revealing the data potential law. However, in the application process, the rough set theory limits the development of this method due to its strict equity. So for this problem Dubois and Prade proposed the concept of fuzzy rough set as a fuzzy promotion of rough sets. Instead of exact collection with a blur collection, introducing a fuzzy similar relationship replaces the precise similar relationship, and expands the basic rough set to a fuzzy rough set. Current fuzzy rough sets can be used in multiple fields, such as determining fitting models based on feature selection and for securities price forecasting.

As the volume of the transaction is related to the index of the share price, if the correlation is greater than the index correlation, the transaction data will generate dramatic fluctuations, so it will result in the direct use of the volume value calculation. The big error cannot truly reflect the relationship between the transaction volume and the stock price, so this paper divides the transaction volume data by introducing the fuzzy rough set theory, dividing the value range of the indicator into several fuzzy rough sets, and determining the input function mapping between output data.

First, the transaction volume data is a blurred segment, and then the determination of the determined function mapping is obtained according to the fuzzy rough set. After the above calculations, the subunities maps available herein are as follows:

$$K(v(t)) = \begin{cases} a, v \leq v_1 \\ \frac{b \cdot v^2}{v_2 - v_1}, v_1 < v < v_2 \\ c, v \geq v_2 \end{cases} \tag{9}$$

where a, b and c are the minimum and maximum values of the map parameters and are the indicators, as a function turning point. For transaction volume data, the transaction volume of the two ends is larger than the fluctuation of the intermediate region, so that the data between the two ends is given the value  $A = 0.1$ ,  $c = 1.0$ . The parameter  $b$  of the intermediate region is determined based on the bias direction and the mean  $\mu(v)$  of each stock, and when the data are biased  $v_2$ ,  $b = \min\{\mu(v) - 0.1, 1.0 - \mu(v)\}$ , when the data is biased  $v_1$ ,  $b = \max\{\mu(v) - 0.1, 1.0 - \mu(v)\}$ , so that the overall data are more compact, so as to avoid the transaction amount due to other related indicators Fluctuations are caused to the stock price.

Through the above introduction, the MFR–GEP algorithm can be described as follows.

Enter: Data Set  $t, x(t), x'(t), \dots, x^{(n-1)}(t), x^{(n)}(t)$ ;

Function set  $\{+, -, *, /, ^ 2\}$ ;

Head length  $h$ ;

Iterative number  $N$ ;

Output: optimal chromosome;

Begin:

1. Random initialisation group

2.  $i = 0$

3. while ( $i < N$ ) {

4. Calculate the regular item weight coefficient ( $\omega$

5. Calculate the subunities mapping *EMBEDEquation.DSMT4*.

7. Retains current algebraic fitness function  $F(t)$  excellent chromosomes for selection

8. Genetic operation on the population (variation, string, recombination)

9.  $i = i + 1$  }

10.end while

11.return fitness function  $F(t)$  for optimum chromosomes

End

### 2.2.3 Higher-order ordinary differential equation model solution

Direct solution of higher-order ordinary differential equations is a complex and difficult problem, using the fourth-order Lunge–Kutta method to transform it into multiple first-order ordinary differential equations before solving [12].

First, convert the formula as follows:

$$(x(t), x'(t), \dots, x^{(n-1)}(t)) = (y_1(t), y_2(t), \dots, y_n(t)) \tag{10}$$

Formula (4) transforms the following system of equations:

$$\begin{cases} y_1'(t) = y_2(t) \\ y_2'(t) = y_3(t) \\ \vdots \\ y_{n-1}'(t) = y_n(t) \\ y_n'(t) = f(t, y_1(t), y_2(t), \dots, y_n(t)) \end{cases} \tag{11}$$

The initial value is

$$y(t_0) = (y_1(t_0), y_2(t_0), \dots, y_n(t_0)) \tag{12}$$

Thus, the above system of equations are solved and a set of prediction values are obtained after several iterations

$$(y_1(t+1), y_1(t+2), \dots, y_1(t+m))$$

That is, the stock price forecast of  $m$  time nodes is

$$(x(t+1), x(t+2), \dots, x(t+m))$$

### 3 Simulation experiment and result analysis

#### 3.1 Data selection and experimental parameter setting

This paper selects the closing price data of all 10 stocks, including YTO Express and Kunlun Wanwei, among which the number of training sets is 118 and the number of test sets is 61. Data experiments are conducted using the MFR–GEP algorithm to predict the stock price for the next 5 days and compare with the standard GEP algorithm and the predictions of the neural network and ARIM algorithms. The experimental parameters are set as shown in Table 1.

**Table 1** Parameter setting table

Parameter Name	Parameter declaration
Dgeneration times	20000
Function set	
{+ - * / ^2}The Symbol Set	
Chromosomes number	30
Number of genes	3
Connection function	+
Head length	8
Variability rate	0.044
IS string rate	0.1
RIS insertion rate	0.1
Gene insertion rate	0.1
Single-point reorganisation rate	0.3
Two-point reorganisation rate	0.3
Gene recombinant rate	0.1

For the prediction results, the average relative error MRE is used as the evaluation criterion. The MRE calculation formula is as follows:

$$MRE = \frac{1}{m} \sum_{i=1}^m \frac{|\hat{y}_i - y_i|}{y_i} \tag{13}$$

where  $m$  is the total number of predicted values,  $\hat{y}_i$  or a forecast value and  $y_i$  or the actual value.

#### 3.2 Experimental Simulation

First, we give the correlation coefficient between the closing price index of 10 stocks. From the coefficient, there is a certain correlation between the transaction volume and the price of the stock. First, according to the correlation coefficient of the stock price and trading volume given, the mean and variance of the corresponding trading volume and stock price data of each stock are calculated, then the amount of information contained by the two indicators are calculated according to Formulas (6) and (7), and finally calculate the weight coefficient using Formula (8), representing the magnitude of the influence of the stock trading volume on the stock price.



Then the subfunction map corresponding to each stock is calculated by Formula (9) for the complete fitness function.

Predicting 10 stocks is done by using this method and traditional stock prediction methods to obtain the average relative error of different prediction methods. Except for the stock of Taiyuan Heavy Industry, the results obtained in this method have small average relative error relative to the neural network and ARIMA method, and the prediction results have a higher accuracy. Moreover, due to the stability requirements of the time series data and neural network, the prediction error of the two methods is relatively unstable, which also reflects the effectiveness and stability of the present algorithm from the side. In the error comparison of this algorithm and the standard GEP algorithm, the relative error of this method is smaller, and this algorithm improves the prediction accuracy by adding the turnover index as the constraint on the stock price.

For the stock of Taiyuan Heavy Industry, the average relative error obtained by the neural network is smaller, but the error value obtained by the method is not much different from it. Therefore, the model of the stock and the forecast value comparison map are given, and the images analyse the results to illustrate the accuracy of the method. For the Taiyuan stock, the functional model obtained by this method is as shown in Formula (14)

$$x'''(t) = t + a_0 \cdot x'(t) \cdot x''(t) \cdot (a_1 x''(t) + x'(t)) + \frac{(2t + a_2) \cdot x'(t)}{(a_3 \cdot x(t))^3} \tag{14}$$

The parameters are:  $[a_0, a_1, a_2, a_3] = [-17.17, 0.20, 11.82, -7.85]$

Judging from Figure 1, the predicted value of the first node obtained by this method is closer to the actual value. Although the average error of the neural network is smaller, the predicted value fluctuation of the neural network changes very small, which is basically in a downward state all the time, and the actual value of the change trend cannot be completely predicted. The predictive value curve of this method is more similar to the actual value curve, and the trend and fluctuation characteristics are the same, which is one of the advantages of the present method, while the error accuracy is within the acceptable range. Thus, it can be reflected that the present paper method has a higher accuracy and the accuracy of the trend prediction.

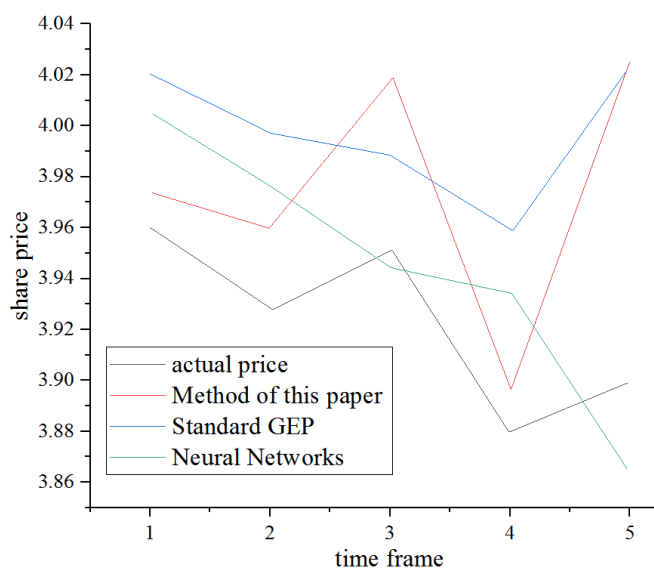


Fig. 1 Comparison of Forecast Results of Taiyuan Heavy Industry



## 4 Conclusion

For the financial stock price, the paper studies the ordinary differential equation, solves the method and the application, and proves the feasibility and effectiveness of the method in financial investment.

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