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Application of Higher-Order Ordinary Differential Equation Model in Financial Investment Stock Price Forecast

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Abstract

In order to improve the efficiency of dynamic system prediction modelling, this paper proposes a predictive model based on high-order normal differential equations to obtain an explicit model. The high-order constant differential equation model is reduced, and the numerical method is used to solve the predictive value. The results show that the method achieves the synchronisation of model establishment and parameter optimisation, in addition to greatly enhancing the modelling efficiency.

Keywords: High-order constant differential equation model, dynamic system modelling, financial investment, stock price
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1 Introduction

Dynamic systems, such as temperature changes, precipitation, financial data change, and so on, vary with time in daily life. Modelling the prediction of dynamic systems with time has always been a research hot spot. An appropriate time series model is of great significance for assessment of investment risk controlling investment output.

Time series prediction is a method for building a model based on the regular information of existing data, and a model is introduced to complete the prediction method. The prediction effect is mainly affected by the model, which is because time series data constitutes a non-regular dynamic system. The data is complicated with time volatility, and the different models have a great difference between the processing of data, building contacts and regular discovery. The model has a different degree of deviation from the description of historical

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data, which in turn has a direct impact on the prediction. For dynamic system sampling of data over time, the autoregressive integrated moving average (ARIMA) model is usually applied with the artificial neural network (ANN) model. The ARIMA model is a classic linear model that poorly handles non-linear problems. In order to improve its non-linearity, the literature combines the ARIMA model with the deep belief network, support vector machine and generalized autoregressive conditional heteroskedasticity (GARCH) models, and it has made a certain amount of red tide forecast, uranium price prediction, network traffic forecasting and subway passenger short-term forecasting [1].

At the end of the 17th century, the sub-division approach was accompanied by the development of calculus, born due to its integrity and ease of operation, so that it has quickly become a powerful tool for studying natural science. Scientists discovered that the actual engineering issues in nature can be used to establish a sub-equation model with initial value and boundary conditions, such as the speed resort differential model established by solving the fastest drop, the Malthus population model and the logistic model established by the population forecast; the non-uniform beam's horizontal vibration, the sixth-order, eighth-order, tenth-order normal differential equation model of the ring structure vibration problem, and so on. During this period, scientists established a large number of solutions to the equation; however, solving these models is an urgent need. The simple model is also good and can be accurately solved using the direct integral method, separation variable method, and so on; however, most models in real life cannot give precise solutions due to the particularity of their physical background complexity and boundary problems. Therefore, it has spurred scientists to study the solution to the solution of the problem from other aspects. Since there is no exact solution of the sub-partition, some scientists have begun to use an approximation to solve it. Based on this idea, the numerical solution of differential equations has become branched, and then it rapidly developed and became a hot topic in the field of mathematics research [2].

Khachay solved the boundary value problem of equations based on Meyer. In the utilisation of many solutions of solutions, many approaches are favoured by many scholars due to the simple forms of solutions. Efendiev studied the Haar function vector and established a Haar wavelet integrated calculator matrix to provide the basis for using the Haar wavelet solution differential equation [4]. Bagd applied the Haar wavelength division operator matrix to the power system problem and promoted the application of wavelet in power systems [5]. Xie used the Haar wavelet method to solve the linear sonocity division, non-linear sub-division, high-order differential equations, one-dimensional diffusion equation, two-dimensional Poisson equation, and so on, as well as a change in the variable steps of the wavelet method [6]. Cooperation extends the Haar wavelet configuration method to linear integral equations, second types of Freholm non-linear integral equations and numerical methods for non-linear solution equations [7]. In China, Karaman used Haar to solve the wavelet number of wavelet mean for wave equations [8]. Kumar obtained a numerical solution of the development equation with Haar slopes [9]. Kennedy used Haar to apply the eigenvalues of high-order differential equations and three-dimensional Parsi equations and three-dimensional double-tuning and equations on the formal area [10].

2 Model establishment and solving based on high-order normal differential equations

2.1 Higher-order ordinary differential equation model

In the case of known historical data, we calculate the differential differences according to the central difference in the literature; then, we establish the function relationship between the differential value of the next time node and the historical data difference value, such as in Eq. (1):

$$\begin{cases} x^{(n)}(t) = f\left(t, x(t), x'(t), ..., x^{(n-1)}(t)\right) \\ x^{(i)}(t_0) = x^{(i)}(0) \end{cases} \tag{1}$$

Among them, T is the time node, $X(t)$ represents the stock clip price of the T node, and $x'(t), \ldots, x^{(n-1)}(t)\, x^{(n)}(t)$ are the differential differences.

With the gene expression programming (GEP) algorithm, the display expression of the high-order alternative equation model of each stock can be obtained for subsequent analysis. At the same time, in order to achieve the goal of utilisation of multi-factor prediction, on the basis of standard GEP, other indicators affecting the stock price change are added to the adaptive function, and finally the high-order regular differential equation model based on multi-factor regularisation (MFR)–GEP algorithm is obtained.

In the evolutionary algorithm, the adaptation function is the main indicator described in the individual performance, guiding the evolutionary direction, which can affect the convergence speed of the algorithm and whether the optimal solution can be found. Different complex systems correspond to different adaptive functions; for the stock system, simple assessment is evaluated as adapted, which is easy to cause the predicted effect, and the error is large. And the stock price is affected by many factors, and different indicators have different effects on the stock price. Therefore, this paper improves the adaptation function, joining the impact indicator, and constrains the share price as a regular item.

2.1.1 Improvement of the adaptation function

The standard regularisation theory only involves linear problems, adding constraints for experience error functions, constrained as a priori knowledge, playing a guiding role, tending to select the direction of gradient decrease in constraints in the process of optimising the error function, so that the ultimate prior knowledge is solved. Simply put, regularisation thinking is to find an approximate solution close to the precise solution, to make it as close as possible.

Since the volume of the transaction is one of the indicators of stock assessment, there is a certain degree of influence on price fluctuations, and this function will be added to the GEP algorithm as a regular item, and the standard GEP is improved.

Because the amount of the volume and the closing price are large, it is not convenient for data analysis; so the transaction amount indicator must first be standardised, and the calculation is made to the interval [0, 1] using Eq. (2).

$$v(t) = \frac{volume(t) - volume_{\min}(t)}{volume_{\max}(t) - volume_{\min}(t)} \tag{2}$$

Where:

$$p(t) = x^{(n)}(t)\, K(v(t)) \tag{3}$$

And the regular item is $\omega \sum |p(t) - p(t-1)|$, $x^{(\hat{n})}(t)$ is the nth-order value of the predicted stock price, $x^{(n)}(t)$ is the Nth-step Nag value of the actual stock price; ω is the weight coefficient of the regular item, reflecting the degree of influence of the transaction index on the stock price; $K(V(t))$ is the mapping for the subunits. For the problems investigated in this article, the specific the value is, the better is the result.

The above improvement adaptation function, based on the standard GEP's adaptivity function, the absolute error function, joining the transaction index as the regular item, constrained the stock price forecast, avoiding a large error in using a single indicator prediction. At the same time, the enhancement algorithm jumps out of local optimal capabilities and improves prediction accuracy. For the calculation of the regular item parameters, this paper uses the correlation between the indicators to determine the weight coefficient and then determines the subunits in the adaptive function based on the basic theory of the fuzzy rough set. We use improved adaptation functions to measure the advantages and disadvantages of the model while increasing the accuracy of data prediction [11].

2.2 Solution of high-order normal differential equation model

2.2.1 Determination of weight ω

Numerous factors influence stock prices, and each indicator is different from the size of the stock price. It is different from the correlation between the stock prices, so the weights of each indicator should also be different. This article has the following solving method for the weight factor of the regular item in the adaptive function.

Suppose A_j indicates the amount of information included in the jth indicator, i.e. the differentiated information Z_j is expressed, the correlation coefficient between the jth indicator and other indicators is r_{jk} and the calculation formulas of r_{jk} and Z_j are known, as shown in Eqs (4) and (5):

$$r_{jk} = \frac{\sum (X_j - \bar{X}_j)(X_k - \bar{X}_k)}{\sqrt{\sum (X_j - \bar{X}_j)^2 \sum (X_k - \bar{X}_k)^2}} \tag{4}$$

$$j = 1,2,\ldots,l, k = 1,2,\ldots,l$$

$$Z_j = \frac{S_j}{X_j}, \quad j = 1,2,\ldots,l \tag{5}$$

$$\mu = \bar{X}_j = \frac{1}{N}\sum_{p=1}^{N} x_{pj}, \quad S_j^2 = \frac{1}{N-1}\sum_{p=1}^{N}(X_{pj} - \mu)^2.$$

Then, A_j can be represented as

$$A_j = Z_j \sum_{k=1}^{l} \left(1 + r_{jk}^2 + \frac{r_{jk}^2}{Z_j - r_{jk}}\right), \quad j = 1,2,...,l \tag{6}$$

Among them, the larger the A_j, the greater the amount of information contained in J IT; the greater the importance of this indicator, the greater the weight and so the weight of JU JII (7):

$$\omega_j = \frac{A_j}{\sum_{j=1}^{l} A_j} \tag{7}$$

In this article, the two indicators selected are daily stock closing prices and daily transactions. Therefore, the weight coefficient of the regular item is $\omega = \omega_2 : \omega_1$.

Thus, by equating Eq. (3)~Eq. (6), the transaction amount indicator is quantified for the importance of the stock price, and the weight coefficient value of the regular item is given to the size of the influence on the stock price, which can effectively reduce the effects of extreme values, making the calculation results more reasonable and reliable.

2.2.2 Determination of sub-function k $(V(t))$

The fuzzy set theory was proposed by US computer experts in 1965 and the rough set theory was proposed by Poland mathematician Pawlak in 1982 and is a method of revealing potential laws. However, in the application process, the rough set theory limits the development of this method due to its strict equity. So, for this problem, Dubois and Prade proposed the concept of fuzzy rough set as a fuzzy promotion of rough sets. Instead of exact collection with a blur collection, introducing a fuzzy similar relationship replaces the precise similar relationship and expands the basic rough set to a fuzzy rough set. Current fuzzy rough sets can be used in multiple fields, such as for determining fitting models based on feature selection, for securities price forecasting, and so on.

Since the volume of the transaction is related to the index of the share price, if the correlation is greater than the index correlation, the transaction data will generate dramatic fluctuations; so, the direct use of the volume value in calculation will result in a big error, which cannot truly reflect the relationship between the transaction volume and the stock price. So, this paper divides the transaction volume data by introducing the fuzzy rough set theory, dividing the value range of the indicator into several fuzzy rough sets, and determines the input function mapping between output data.

First, the transaction volume data is a blurred segment, and then the determination of the function mapping is conducted according to the fuzzy rough set. After the above calculations, the subunit maps available herein are as follows:

$$\mathrm{K}(v(\mathrm{t})) = \begin{cases} a, v \leq v_1 \\ \frac{\mathrm{b}v^2}{v_2 - v_1}, v_1 < v < v_2 \\ c, v \geq v_2 \end{cases} \tag{8}$$

Here, A, B and C are the minimum and maximum values of the map parameters, v_1 and v_2 are the indicators, functioning as a turning point. For transaction volume data, the transaction volume of the two ends is larger than the fluctuation of the intermediate region, so that the data between the two ends is given the following values: i.e. $A = 0.1, C = 1.0$. The parameter B of the intermediate region is determined based on the bias direction and the mean $\mu(v)$ of each stock; when the data is biased with respect to v_2, $\mathrm{b} = \min\{\mu(v) - 0.1, 1.0 - \mu(v)\}$; when the data is biased with respect to v_1, $\mathrm{b} = \max\{\mu(v) - 0.1, 1.0 - \mu(v)\}$, so that the overall data is more compact, so as to avoid the transaction amount fluctuations caused to the stock price due to other related indicators.

Through the above introduction, the MFR–GEP algorithm can be described as follows.

Enter: Dataset $\mathrm{t}, \mathrm{x}(\mathrm{t}), \mathrm{x}'(\mathrm{t}), \cdots, \mathrm{x}^{(\mathrm{n}-1)}(\mathrm{t}), \mathrm{x}^{(\mathrm{n})}(\mathrm{t})$;
Function set $\{+, -, *, /, \wedge 2\}$;
Head length h;
Iterative number N;
Output: optimal chromosome;
Begin:
1. Random initialisation group
2. $i = 0$
3. while (i<N) {
4. Calculate the regular item weight coefficient ω
5. Calculate the subunit mappings $\mathrm{K}(v(\mathrm{t}))$
7. Retains current algebraic fitness function $F(t)$ excellent chromosomes for selection
8. Genetic operation on the population (variation, string, recombination)
9. $i = i + 1\}$
10. End while
11. Return fitness function $F(t)$ for optimum chromosomes
End

2.2.3 Higher-order ordinary differential equation model solution

Direct solution of higher-order ordinary differential equations is a complex and difficult problem; we use the fourth-order Longge-Kuta method to transform it into multiple first-order ordinary differential equations before solving [12].

First, convert the formula as follows:

$$\left(\mathrm{x}(\mathrm{t}), \mathrm{x}'(\mathrm{t}), \cdots, \mathrm{x}^{(\mathrm{n}-1)}(\mathrm{t})\right) = (\mathrm{y}_1(\mathrm{t}), \mathrm{y}_2(\mathrm{t}), \cdots, \mathrm{y}_\mathrm{n}(\mathrm{t})) \tag{9}$$

Equation (3) transforms the following system of equations:

$$\begin{cases} y_1'(t) = y_2(t) \\ y_2\prime(t) = y_3(t) \\ \vdots \\ y_{n-1}\prime(t) = y_n(t) \\ y_n\prime(t) = f(t, y_1(t), y_2(t), \cdots, y_n(t)) \end{cases} \tag{10}$$

The initial value is

$$y(t_0) = (y_1(t_0), y_2(t_0), \cdots, y_n(t_0)). \tag{11}$$

Thus, the above system of equations is solved and a set of prediction values are obtained after several iterations:

$$(y_1(t+1), y_1(t+2), \cdots, y_1(t+m))$$

That is, the stock price forecast of *m* time nodes is

$$(x(t+1), x(t+2), \ldots, x(t+m))$$

3 Simulation experiment and result analysis

3.1 Data selection and experimental parameter setting

This paper selects the closing price data of all 10 stocks, including YTO Express and Kunlun Wanwei, among which the number of training sets is 118 and the number of test sets is 61. Data experiments are conducted using the MFR–GEP algorithm to predict the stock price for the next 5 days and compare with the standard GEP algorithm and the predictions of the neural network and ARIMA algorithms. The experimental parameters are set as shown in Table 1.

Table 1 Parameter settings

Parameter name	**Parameter declaration**
Generation times	20,000
Function set	$\{+, -, -, /\wedge 2\}$
The symbol set	$\{t, x(t), x'(t), x''(t)x''''(t), c\}$
Chromosome number	30
Number of genes	3
Connection function	+
Head length	8
Variability rate	0.044
IS string rate	0.1
RIS insertion rate	0.1
Gene insertion rate	0.1
Single-point reorganisation rate	0.3
Two-point reorganisation rate	0.3
Gene recombination rate	0.1

RIS, root insertion sequence.

For the prediction results, the average magnitude of relative error (MRE) is used as the evaluation criterion. The MRE calculation formula is as follows:

$$\mathrm{MRE} = \frac{1}{m}\sum_{i=1}^{m}\frac{|\hat{y}_i - y_i|}{y_i}, \tag{12}$$

where *m* is the total number of predicted values, $\hat{y}_i$ for a forecast value, y_i for the actual value.

3.2 Experimental simulation

First, we give the correlation coefficient between the closing price indexes of 10 stocks. From the coefficient, there is a certain correlation between the transaction volume and the price of the stock. First, according to the correlation coefficient of the stock price and trading volume given, the mean and variance of the corresponding trading volume and stock price data of each stock are calculated; then, we calculate the amount of information contained by the two indicators according to Eqs (5) and (6) and, finally, we calculate the weight coefficient using Eq. (7), representing the magnitude of the influence of the stock trading volume on the stock price. Then, the sub-function map corresponding to each stock is calculated using Eq. (8) for the complete fitness function.

Predicting 10 stocks using this method and traditional stock prediction methods, we obtain the average relative error of different prediction methods. Except for the stock of Taiyuan Heavy Industry, the results obtained by this method have small average relative error relative to the neural network and ARIMA methods, and the prediction results have higher accuracy. Moreover, due to the stability requirements of time series data and neural networks, the prediction error of the two methods is relatively unstable, which also reflects the effectiveness and stability of the present algorithm. In the error comparison of this algorithm and the standard GEP algorithm, the relative error of this method is smaller, and this algorithm improves the prediction accuracy by adding the turnover index as the constraint on the stock price.

For the stock of Taiyuan Heavy Industry, the average relative error obtained by the neural network is smaller, but the error value obtained by this method is not much different from it. Therefore, the model of the stock and the forecast value comparison map are given, and the images are analysed to illustrate the accuracy of the method. For the Taiyuan stock, the functional model obtained by this method is shown in Eq. (13)

$$x'''(t) = t + a_0 x'(t)\, x''(t)\left(a_1 x''(t) + x\prime(t)\right) + \frac{(2t + a_2)\, x\prime(t)}{a_3 x(t)^3} \tag{13}$$

The parameters are as follows: $[a_0, a_1, a_2, a_3] = [-17.17, 0.20, 11.82, -7.85]$.

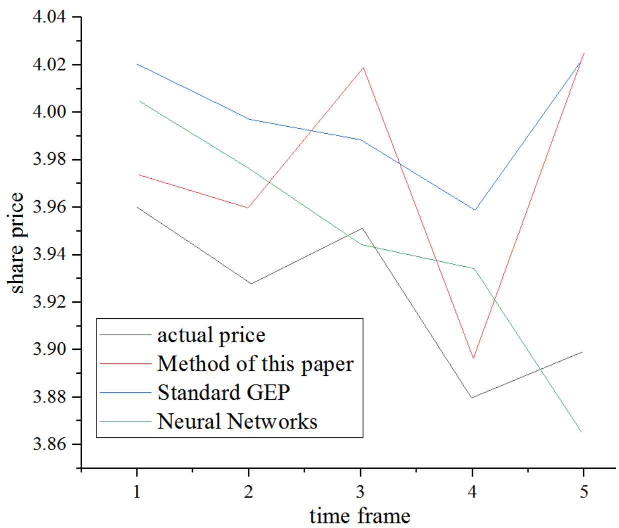

Fig. 1 Comparison of forecast results of Taiyuan Heavy Industry

Judging from Figure 1, the predicted value of the first node obtained by this method is closer to the actual value. Although the average error of the neural network is smaller, the predicted value fluctuation of the neural network is very small, which is basically in a downward state all the time, and the actual value of the change trend cannot be completely predicted. The predictive value curve of this method is more similar to the actual value curve, and the trend and fluctuation characteristics are the same, which is one of the advantages of the present method, while the error accuracy is within the acceptable range. Thus, it can be concluded that the herein-presented method has a higher accuracy and accurate trend prediction.

4 Conclusion

For the financial stock price, the paper applies the ordinary differential equation, solves the method, shows its application and proves the feasibility and effectiveness of the method in financial investment.

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