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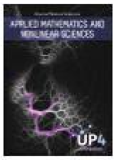

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The Effect of Children's Innovative Education Courses Based on Fractional Differential Equations

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Abstract

Fractional differential equations are one of the important contents of advanced mathematics courses. The article uses fractional differential equations to describe the effects of children's innovative education courses. Through the qualitative analysis of the basic model, several conditions to ensure the effect of children's innovative education courses are obtained. At the same time, combined with practical experience, the teaching curriculum case design analyzes the specific application of the fractional differential equation in the effect of children's innovative education curriculum. Research has found that the fractional differential equation algorithm improves the efficiency of innovation.

Keywords: Children's innovative education; Educational fees; Differential equation model; Educational cost-sharing
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1. Introduction

Higher education fees are a major matter related to the interests of the country, higher education institutions, educated persons, and their families. Over the years, several educational theory scholars have conducted useful discussions on this from academic economics, sociology, and law perspectives and put forward some corresponding countermeasures [1]. In this article, our main purpose is to put forward two differential equation models for higher education fees. At the same time, we will further study the macro-control issues of higher education fees and enrollment scale.

2. Two basic differential equation models for college education fees

We set the number of enrollment of colleges and universities N as a continuous variable, and the education charge of colleges and universities is $R(N)$ [2]. The education cost-shared per capita is $\gamma(N)$. The unpaid expenses of poor students are $p(N)$. They are all functions of enrollment. We get the following differential equation model (I) to describe the problem of college education fees

$$R' = \delta\gamma(N) - \sigma p(N) \tag{1}$$

Where $\delta > 0, \sigma > 0$ is the scale factor. Since the per capita unpaid expenses of poor college students $p(N)$ will increase with the increase in the per capita education cost $\gamma(N)$ shared by the individual educated [3].

$$R' = \delta\gamma(N) - \sigma g(\gamma(N)) \tag{2}$$

Assuming that $p(N)$ determines $\gamma(N)$ and $p(N)$, then we have $\gamma(N) = f(R(N))$ and $p(N) = h(R(N))$ respectively. Where $f(R)$ and $h(R)$ are both monotonically increasing functions [4]. Assuming $f'(R) \geq 0, h'(R) \geq 0$:

$$R' = \delta f(R) - \sigma h(R) \tag{3}$$

In the same way, we can also obtain (2) the corresponding differential equation model (Ⅳ) describing the issue of college education charges

$$R' = \delta f(R) - \sigma g(f(R)) \tag{4}$$

If $N, R, f(R)$ and $h(R)$ in the model (3) are regarded as time, population size, absolute reproduction rate, and death rate, respectively, then what (3) describes is exactly the different functional forms of the differential equation model of the free development of the population. Because there is no direct connection between mortality and reproduction rate in the population development model, (2) and (4) are the main governing equation models proposed in this paper for the issue of college education fees.

3. Qualitative analysis of the basic model of higher education fees

In this section, we mainly use qualitative analysis (2) and (4) to seek solutions to the problem of college education fees.

$$\delta\gamma(N) = \sigma g(\gamma(N)) \tag{5}$$

This shows that a sufficient and necessary condition for universities to keep their education fees unchanged is to balance the per capita education costs shared by individuals and the unpaid expenses $g(\gamma(N))$ per capita by poor students. It is required that equation (5) has a positive root N_0, so that the number of college enrollment N_0 and the per capita education cost $\gamma_0 = \gamma(N_0)$ shared by individuals should be determined [5]. Therefore, under the condition of expanding the enrollment scale of colleges and universities, increasing government investment in education funds from policy measures, preferential incentives, and attracting social donations to run schools can solve the problem. At the same time, student loan scholarship, work-study, and other forms to solve the ability of poor students to pay are all solutions to achieve a balance.

(1) When $\gamma'(N_0)>0, 0<g'(\gamma_0)<\dfrac{\delta}{\sigma}$, the education fees of $R'(N_0)=0, R''(N_0)>0$ colleges and universities reach a minimum. This shows that under the condition that the per capita education cost $\gamma(N)$ shared by individuals increases, as long as the growth rate $g'(\gamma)$ of unpaid expenses per poor student is controlled within a certain limit, the minimum fee $R(N_0)$ of colleges and universities can be kept unchanged.

(2) When $\gamma'(N_0)<0, g'(\gamma_0)>\dfrac{\delta}{\sigma}$, there is $R'(N_0)=0, R''(N_0)>0$ college education fees that reach a minimum. This shows that if the growth rate of the per capita unpaid expenses of poor students exceeds a degree $\dfrac{\delta}{\sigma}$, then only by reducing the per capita education cost $\gamma(N)$ shared by individuals can the minimum fee $R(N_0)$ of colleges and universities remain unchanged.

(3) When $\gamma'(N_0)>0, 0<g'(\gamma_0)<\dfrac{\delta}{\sigma}$, the education fees of $R'(N_0)=0, R''(N_0)<0$ colleges and universities reach a maximum value. This shows that when the per capita education cost $\gamma(N)$ shared by individuals is reduced, and the growth rate of unpaid expenses per capita of poor students is within a certain limit $\dfrac{\delta}{\sigma}$, colleges and universities can keep their extremely high fees $R(N_0)$ unchanged.

(4) Under the condition $\gamma'(N_0)>0, g'(\gamma_0)>\dfrac{\delta}{\sigma}$, $\gamma(N)$ monotonously increases and $g'(\gamma)$ is also large, at this time the education fees of colleges and universities reach a maximum value. High fees will cause some poor students to owe more tuition fees or drop out of school due to their inability to pay. From model (4), it can be seen that the necessary and sufficient condition for the school's education charge $R(N)$ to be constant is

$$\delta f(R)=\sigma f(f(R)) \tag{6}$$

This requires (6) there is a positive root R_0, and there is also $\gamma_0=f(R_0)$. Because R_0 is the equilibrium point of (4), we have the following stability conclusions about education fees R_0 according to the stability judgment method.

(5) When $0<g'(\gamma_0)<\dfrac{\delta}{\sigma}, f'(R_0)>0$, R_0 is the stable equilibrium point of the model (4). This shows that under the condition that the per capita education cost $\gamma=f(R)$ shared by individuals increases, only when the growth rate of unpaid expenses per poor student is within a certain limit $\dfrac{\delta}{\sigma}$ will the education fees R_0 of colleges and universities remain stable.

(6) When $g'(\gamma_0)>\dfrac{\delta}{\sigma}, f'(R_0)<0$, R_0 is also the stable equilibrium point of the model (4). This shows that if the growth rate of unpaid expenses per capita of poor students exceeds a degree $\dfrac{\delta}{\sigma}$, then only by reducing the per capita education cost-shared by individuals can the education fees R_0 of colleges and universities remain stable.

(7) The school's education fees R_0 are unstable under other conditions.

4. Several special models and macro-control analysis

In free education $(\gamma = 0)$, the number of students is n. In charging education, the per capita education cost that the government stipulates that the individual shares is γ_M is the number of students enrolled in colleges and universities is $M > n$. We will discuss in three situations.

4.1 The per capita education cost-shared by individuals increases with the increase in the number of students enrolled

Under the condition that individuals' per capita education cost increases with the increase in the number of enrolled students, we set $\gamma(N) = a + bN$ and $g(\gamma) = g_0\gamma$ as linear functions in a model (2). So we have $\gamma(N) = b(-n+N)$ where $b = \dfrac{\gamma_M}{M-n} > 0$. After substituting (2), we get the following education fee model

$$R(N) = b(\delta - \sigma g_0)(-n+N) \tag{7}$$

At $N > n$, the equilibrium condition for keeping the higher education fees unchanged is $\delta = \sigma g_0$. Because of $0 < \gamma(N) \le \gamma_M$, so $n < N \le M$. The enrollment scale of this university can be expanded within the scope prescribed by the government. Under non-equilibrium conditions, when $g < \dfrac{\delta}{\sigma}, N > n$ (or $g_0 < \dfrac{\delta}{\sigma}, N > n$), there is G (or $R' < 0$). This shows that when colleges and universities expand their enrollment scale, only when the growth rate of unpaid expenses per capita for poor students is low can they consider raising education fees [6]. We have colleges and universities that charge for education as a function of the number of students enrolled.

$$R(N) = \frac{1}{2} b(\delta - \sigma g_0)(-n < N)^2 \tag{8}$$

It can be seen that the adjustment of education fees according to (8) will be higher. If $g(\gamma) = g_0\gamma^2$ is changed to a quadratic function in a model (2), then it is easy to know that the scale of enrollment of colleges and universities whose education fees are positive is

$$n < N \le n + \frac{3\delta(M-n)}{2g_0\gamma_M} \tag{9}$$

It can be seen that when the government stipulates that the per capita education cost to be shared is lower ($\gamma_M \to 0$), it is conducive to colleges and universities to expand the scale of independent enrollment. We then proceed from the model (4) for qualitative analysis. Assume that $\gamma = f(R) = c + dR$ is a linear function and $g(\gamma) = g_0\gamma^{1+T}$ is a power function. Where $T > 0$. Because there is $R = 0, \gamma = 0$ in free education. In charging education, if the government-specified education fee is R_0, the per capita education cost-shared by individuals is γ_0. Then we have $\gamma = dR, g(\gamma) = g_0 d^{1+T} R^{1+T}$. Where $d = \dfrac{\gamma_0}{R_0} > 0$. After substituting (4), we get the following education fee model

$$R' = \delta dR - \sigma g_0 d^{1+T} R^{1+T} \tag{10}$$

Model (10) has an unstable equilibrium point $R_1 = 0$ and a stable equilibrium point $R_2 = \frac{1}{d}^T \sqrt{\frac{\delta}{\sigma g_0}}$. When $T = 1$ is taken, the model (10) is a Logistic model, and there is $\frac{R_2}{R_0} = \frac{\delta}{\sigma g_0 \gamma_0}$ at this time [7]. So there is $R_2 > R_0$ when $0 < \gamma_0 < \frac{\delta}{\sigma g_0}$ is. This shows that when individuals' per capita education cost is within a certain limit, the stable education fees of colleges and universities will increase. When $\gamma_0 > \frac{\delta}{\sigma g_0}$, there is $R_2 > R_0$. This shows that when individuals' per capita education cost-shared exceeds one degree, the phenomenon of some students in arrears in tuition fees will reduce the stable education fees of colleges and universities.

4.2 The per capita education cost-shared by individuals decreases with the increase in the number of students enrolled

We assume in the model (2) that $\gamma(N) = \frac{b}{N}$ and $g(\gamma) = g_0\gamma$ are set up above, and we have $\gamma(N) = \gamma_M \frac{M}{N}$. After substituting (2), the following educational charging model is obtained

$$R'(N) = (\delta - \sigma g_0)\frac{M\gamma_M}{N} \tag{11}$$

Points (11) The relationship between education fees and enrollment is logarithmic

$$R(N) = (\delta - \sigma g_0)M\gamma_M \ln\frac{N}{n} \tag{12}$$

Because of $\ln\frac{N}{n} = o(N) = o(N^2)(N \to \infty)$, our (12) to determine the charge is lower than (8). The educational fees of colleges and universities can increase slowly with the expansion of enrollment scale and then $0 < \gamma(N) \le \gamma_M$ knows $N \ge M$. It can be seen that the enrollment scale of colleges and universities can exceed the government's regulations, and the fees increase more slowly [8]. This is an education fee and enrollment expansion model that is worth promoting. If $g(\gamma) = g_0\gamma^2$ is assumed in model (2), then it is easy to know that the number of students enrolled in universities to maintain an increase in education fees is

$$N \ge \frac{\sigma g_0 \gamma_M M}{\delta} \tag{13}$$

And the education fees of colleges and universities are

$$R(N) = \sigma M\gamma_M \ln\frac{N}{n} - \sigma g_0 M^2\gamma_M^2\left(\frac{1}{n} - \frac{1}{N}\right) \tag{14}$$

When $N \to \infty$, the approximate enrollment scale of the university's education fees is positive is

$$N \ge n\exp\left[\frac{\sigma g_0}{\delta n} M\gamma_M\right] \tag{15}$$

The formula (15) shows that the enrollment of colleges and universities must reach a certain scale to avoid the result of school losses [9].

4.3 The per capita education cost-shared by individuals is a bounded function of the number of students enrolled

We assume $\gamma(N)=\dfrac{bN+c}{1+N}$ and $g(\gamma)=g_0\gamma$ in the model (2). We have a monotone bounded function $\gamma(N)=\dfrac{b(-n+N)}{1+N}$, where $b=\dfrac{\gamma_M(1+M)}{M-n}>0$ is. After substituting (2), the following educational charging model is obtained

$$R'(N)=\frac{\delta b(-n+N)}{1+N}-\frac{\sigma g_0 b(-n+N)}{1+N} \tag{16}$$

Points (16) The relationship between education fees and enrollment is logarithmic

$$R(N)=(\delta-\sigma g_0)b[N-n-(n+1)\ln\frac{N+1}{n+1}] \tag{17}$$

If $g(\gamma)=g_0\gamma^2$ is assumed in model (2), then it is easy to know that when $\delta<\sigma g_0 b$, the enrollment scale for universities to maintain their educational fee increase is

$$n<N\leq\frac{\sigma g_0 bn+\delta}{\sigma g_0 b-\delta} \tag{18}$$

Therefore, under condition $\delta<\sigma g_0 b$, colleges and universities should limit the scale of enrollment.

Finally, we assume $\gamma=f(R)=\dfrac{dR}{1+R}, g(\gamma)=g_0\gamma^2$ in the model (4). If the education fee set by the government is R_0, the per capita education cost-shared by individuals is γ_0, then we have $\gamma=d_1R$. Among them, after $d_1=\dfrac{1+R_0}{R_0}\gamma_0$ is substituted into (4), we get the following logistic model of education charges

$$R'=\delta d_1R-\sigma g_0 d_1^2R^2 \tag{19}$$

When $\delta<\delta g_0 d_1$, equation (19) has unstable education charges

$$R=\frac{\delta R_0}{\sigma R_0(1+R_0)-\delta R_0} \tag{20}$$

5. Extension of the model

5.1 Mixed differential equation model

According to different enrollment scales, colleges and universities consider adopting corresponding education charging models to mix two or more models [10]. We adjust the charging method of a college education according to the number of people

$$\begin{cases} R_1'(N)=b(\delta-\sigma g_0)(-n+N) \\ R_2'(N)=(\delta-\sigma g_0)\dfrac{M\gamma_M}{N} \\ R_1(N_0)=0, R_2(N_1)=R_1(N_1) \end{cases} \tag{21}$$

5.2 Two-dimensional coupled differential equation model

Universities in different (regions) all adopt the same charging model, and there is a certain mutual influence between them [11]. Then we can integrate more than two models. Consider the logistic model of second-order nonlinear coupling:

$$\begin{cases} R_1^{'} = \delta d_1 R_1 - \sigma g_0 d_1^2 R_2^2 \\ R_2^{'} = \delta d_1 R_2 - \sigma g_0 d_1^2 R_1^2 \\ R(N_0) = R_{10}, R_2(N_0) = R_{20} \end{cases} \tag{22}$$

5.3 Three-dimensional differential equation model

Due to the imbalance of economic and social development, different colleges and universities adopt different charging models, which leads to the unfair phenomenon of education fees and the scale of enrollment [12]. If the control variable $a(N)$ that changes with the number of enrolled students is introduced, then we can get a three-dimensional differential equation model.

$$\begin{cases} R_1^{'} = \delta d_1 R_1 - \sigma g_0 a \\ R_2^{'} = \delta d_2 R_2 - \sigma g_0 d_2^2 R_1^2 \\ a = e_1 R_1 + e_2 R_2 \\ R_1(N_0) = R_{10}, R_2(N_0) = R_{20}, a(N_0) = a_0 \end{cases} \tag{23}$$

6. Conclusion

This article mainly proposes two basic differential equation models (2) and (4) to describe the issue of college education fees. By analyzing several specialcoun models (2) and (4) in three situations, some new conclusions and suggestions on macro-control of university education fees and enrollment scale are given.

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